Math 4133 - Linear Algebra

Quiz #7 - 2013.02.01

Solutions

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1. Compute the determinant of the following matrices:

   Each of these are elementary matrices, so the determinants should be easy to compute.

   (a) $E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

   Here, we are simply scalar multiplying a single row of I_4 by 3, thus $\det(E_1) = 3 \cdot \det(I_4) = 3 \cdot 1 = 3$.

   (b) $E_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

   Here, we are swapping two rows of I_4 twice, thus $\det(E_2) = (-1)^2 \cdot \det(I_4) = 1$.

   (c) $E_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 5 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

   Lastly, this elementary matrix adds 5 times row one of I_4 to row three of I_4. This does nothing to the determinant of I_4, so $\det(E_3) = \det(I_4) = 1$.