

Math 1513 - College Algebra

Written Assignment 4 - Due 2011.02.05

Directions: Please answer the following question in complete sentences. Be sure to label all geometric objects in any illustrations. I will accept an answer in a scanned image format, in a Word document or as a pdf.

Question: While it is a simple concept for real numbers, the square root of a complex number is much more involved due to interplay between its real and imaginary parts. For $z = a + bi$ the square root of z can be found using the formula

$$\sqrt{z} = \frac{1}{\sqrt{2}} \left(\sqrt{|z| + a} \pm i \sqrt{|z| - a} \right),$$

where the sign is chosen to match the sign of b . Use the formula to find \sqrt{z} if $z = 4 - 3i$.

First up, notice that $|z| = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$. So now our formula is

$$\sqrt{4 - 3i} = \frac{1}{\sqrt{2}} \left(\sqrt{5 + 4} \pm i \sqrt{5 - 4} \right),$$

which we simplify to

$$\sqrt{4 - 3i} = \frac{1}{\sqrt{2}} (3 \pm i),$$

And since $b < 0$, we use the minus sign, so our final answer is

$$\sqrt{4 - 3i} = \frac{1}{\sqrt{2}} (3 - i),$$

Let us verify that this is correct. So we shall square our answer and see if we get $4 - 3i$ back.

$$\begin{aligned} \left(\frac{1}{\sqrt{2}} (3 - i) \right)^2 &= \frac{1}{2} (3 - i)^2 \\ &= \frac{1}{2} (9 - 6i - 1) \\ &= \frac{1}{2} (8 - 6i) \\ &= 4 - 3i \end{aligned}$$