Math 1613 - Trigonometry

Exam #1 - 2010.08.31

Solutions

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1. Find the supplementary angles with measures $6x - 4$ and $8x - 12$ degrees.

   Supplementary angles must add up to 180°, so we have

$$\begin{aligned} (6x + 4) + (8x - 12) &= 180 \\ 14x - 16 &= 180 \\ 14x &= 196 \\ x &= 14^\circ \end{aligned}$$

   Thus, the measure of the angles are $8(14) - 12 = 100^\circ$ and $6(14) - 4 = 80^\circ$.

2. Find the complementary angles with measures $3x - 5$ and $6x - 40$ degrees.

   Complementary angles must add up to 90°:

$$\begin{aligned} (3x - 5) + (6x - 40) &= 90 \\ 9x - 45 &= 90 \\ 9x &= 135 \\ x &= 15^\circ \end{aligned}$$

   Thus, the measure of the angles are $3(15) - 5 = 40^\circ$ and $6(15) - 40 = 50^\circ$.

3. Perform the following calculations.

   a) $62^\circ 18' - 28^\circ 57'$

$$\begin{aligned} 62^\circ 18' - 28^\circ 57' &= 61^\circ 78' - 28^\circ 57' \\ &= 33^\circ 21' \end{aligned}$$

   b) $26^\circ 20' + 15^\circ 37'$

$$26^\circ 20' + 15^\circ 37' = 41^\circ 57'$$

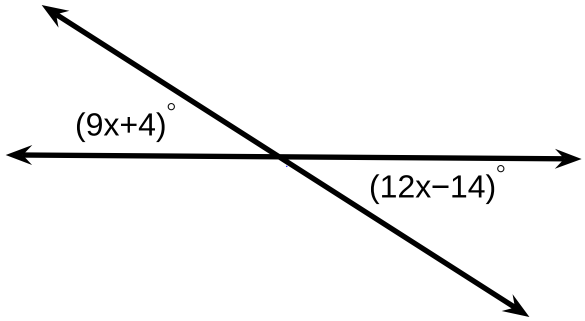
4. Find the angle of least positive measure (not equal to the given measure) coterminal with the angle of -67°.

   To be coterminal, they must terminate at the same point, this our answer should be $-67^\circ + 360^\circ = 293^\circ$.

5. A pulley rotates through 75° in 1 minute. How many rotations does the pulley make in an hour?

$$\frac{75^\circ}{1\text{ min}}\frac{60\text{ min}}{1\text{ hour}}\frac{1\text{ rev}}{360^\circ} = \frac{75 \cdot 60}{360}\frac{\text{rev}}{\text{hour}} = \frac{75}{6}\frac{\text{rev}}{\text{hour}}$$

6. Find the measure of each marked angle.



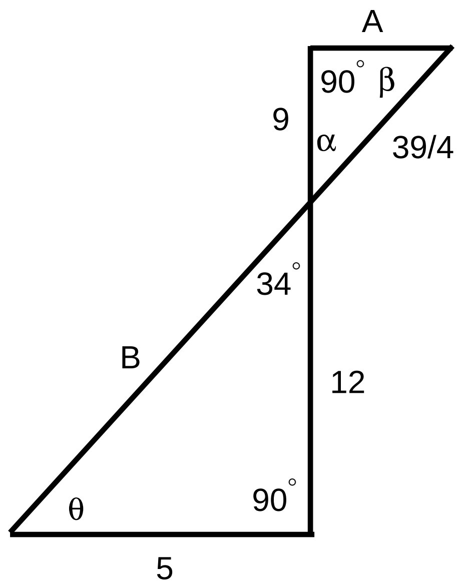
The two angles in question are vertical angles. Therefore we have

$$9x + 4 = 12x - 14$$
$$18 = 3x$$
$$x = \frac{18}{3}$$

and plugging this into the formulas for each angle should give the same result:

$$9\frac{18}{3} + 4 = 58^\circ, \quad 12\frac{18}{3} - 14 = 58^\circ$$

7. Find the measure of the angles α, β, and θ. Find the lengths of sides A and B.



By definition, $\alpha = 34°$, $\beta = \theta = 90° - 34° = 56°$. We now have all the unknown angles. We use ratios from similar triangles for the sides A and B:

$$\frac{12}{5} = \frac{9}{A} \longrightarrow A = \frac{15}{4}$$
$$\frac{B}{12} = \frac{39/4}{9} \longrightarrow B = 13$$

8. Evaluate each of the following trigonometric expressions.

a) $\cos(90°) + 3\sin(270°)$

$$\cos(90°) + 3\sin(270°) = 0 + 3(-1) = -3$$

b) $3\sec(180°) - 5\tan(360°)$

$$3\sec(180°) - 5\tan(360°) = 3(-1) - 5(0) = -3$$

9. Fill in the following table:

	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\csc(\theta)$	$\sec(\theta)$	$\cot(\theta)$
$0°$	0	1	0	undefined	1	undefined
$30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
$45°$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
$90°$	1	0	undefined	1	undefined	1

10. If a building that is 40 ft. tall casts a shadow that is 8 ft. long, and the building across the road casts a 10ft. shadow at the same time, how much taller is this building?

If x is the height of the building across the road, then by similar triangles we have

$$\frac{40}{8} = \frac{x}{10} \longrightarrow x = 50.$$

Therefore, the building across the road is 10 ft. taller.

11. Given $\sin(\theta) = \dfrac{\sqrt{3}}{5}$ with $\cos(\theta) < 0$, find the remaining five trigonometric values corresponding to the given information.

From the information given we are looking at quadrant 2. We can use the Pythagorean Theorem to find the remaining side:

$$\sqrt{3}^2 + x^2 = 5^2 \longrightarrow x = \pm\sqrt{22},$$

where we choose the negative for quadrant 2.

$$\sin(\theta) = \frac{\sqrt{3}}{5}, \cos(\theta) = -\frac{\sqrt{22}}{5}, \tan(\theta) = -\frac{\sqrt{3}}{\sqrt{22}}, \csc(\theta) = \frac{5}{\sqrt{3}}, \sec(\theta) = -\frac{5}{\sqrt{22}}, \cot(\theta) = -\frac{\sqrt{22}}{\sqrt{3}}$$

12. Determine whether each of the following statements are possible.

a) $\tan(\theta) = 0.24$

This is possible since the rang of tan is all real numbers.

b) $\sin(\theta) = 2\pi$

Not possible since $-1 \leq \sin(\theta) \leq 1$.

c) $\csc(\theta) = 0.9$

Not possible since $|\csc(\theta)| \geq 1$.

13. Suppose that $90° < \theta < 180°$, what is the sign of $\sin(2\theta)$?

If $90° < \theta < 180°$, then $180° < 2\theta < 360°$, which corresponds to quadrants 3 and 4, where sin is negative.

14. Find the value of θ such that $\sin(4\theta + 2°)\csc(3\theta + 5°) = 1$.

$$\sin(4\theta + 2°)\csc(3\theta + 5°) = \frac{\sin(4\theta + 2°)}{\sin(3\theta + 5°)} = 1$$

Therefore, we require that $\sin(4\theta + 2°) = \sin(3\theta + 5°)$, or $4\theta + 2° = 3\theta + 5°$. Solving for θ gives $\theta = 3°$.

The End