# Math 1613 - Trigonometry

## Exam #2 - 2010.09.16

## Solutions

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1. Evaluate the following expression.

$$\sec^2(300°) - 2\cos^2(150°) + \tan(45°)$$

$$\begin{aligned}
\sec^2(300°) - 2\cos^2(150°) + \tan(45°) &= \sec^2(60°) - 2\cos^2(30°) + \tan(45°) \\
&= \frac{1}{\cos^2(60°)} - 2\cos^2(30°) + \tan(45°) \\
&= 4 - 2\frac{3}{4} + 1 \\
&= \frac{7}{2}
\end{aligned}$$

2. Find one solution to the equation.

$$\tan(5x + 11°) = \cot(6x + 2°)$$

First we use the cofunction identity for tangent and cotangent:

$$\cot(90° - (5x + 11°)) = \tan(5x + 11°)$$

Therefore, we have

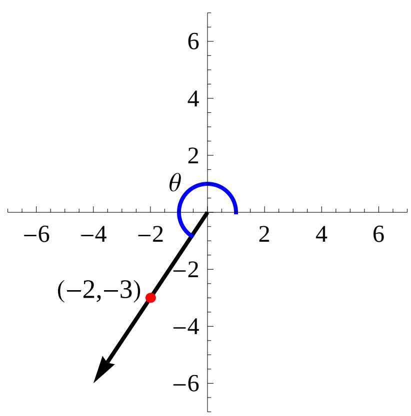
$$\cot(79° - 5x) = \cot(6x + 2°)$$

and we get

$$79° - 5x = 6x + 2°$$

Solving for $x$ gives $x = 7°$.

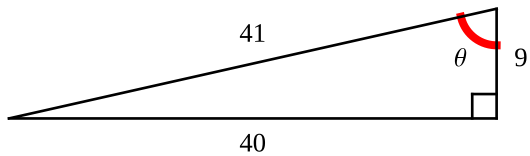
3. Find the sine, cosine and tangent function values for the following angle depicted below.




First we need to compute $r$, which is given by $r = \sqrt{2^2 + 3^2} = \sqrt{13}$.

$$\sin(\theta) = -\frac{3}{\sqrt{13}}, \cos(\theta) = -\frac{2}{\sqrt{13}}, \tan(\theta) = \frac{3}{2}$$

4. Give the six trigonometric function values of angle $\theta$ as depicted in the following figure.




$$\sin(\theta) = \frac{40}{41},\ \cos(\theta) = \frac{9}{41},\ \tan(\theta) = \frac{40}{9}$$
$$\csc(\theta) = \frac{41}{40},\ \sec(\theta) = \frac{41}{9},\ \cot(\theta) = \frac{9}{40}$$

5. Find exact values of the six trigonometric functions for the angle $\theta = -1290°$.

First, note that

$$-1290° = -[360° \cdot 3 + 180° + 30°],$$

and therefore, $-1290°$ is coterminal with $-120°$ which lies in the second quadrant, and has reference angle of $30°$.

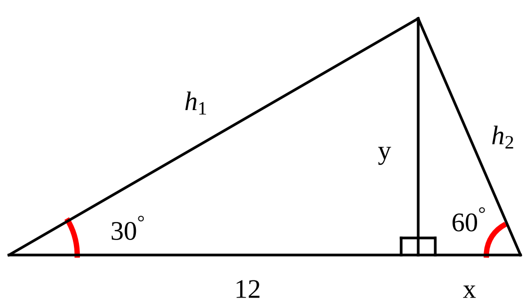
$$\sin(-1290°) = \frac{1}{2},\ \cos(-1290°) = -\frac{\sqrt{3}}{2},\ \tan(-1290°) = -\frac{1}{\sqrt{3}},$$
$$\csc(-1290°) = 2,\ \sec(-1290°) = -\frac{2}{\sqrt{3}},\ \cot(-1290°) = -\sqrt{3}$$

6. Which of the following cannot be exactly determined using reference angles:

a) $\sin(250°)$ b) $\tan(135°)$ c) $\cos(126°)$ d) $\csc(390°)$

Reference angle for b) is $45°$, and reference angle for d) is $30°$. The angles in parts a) and c) do not have corresponding reference angles for which we know the trig values.

7. Find values for the lengths of the unknown sides in the following figure.




First, notice that $\tan(30°) = \frac{y}{12}$, which gives

$$\frac{1}{\sqrt{3}} = \frac{y}{12} \longrightarrow y = \frac{12}{\sqrt{3}}$$

Next, note that $\cos(30°) = \dfrac{12}{h_1}$, and thus

$$\frac{\sqrt{3}}{2} = \frac{12}{h_1} \longrightarrow h_1 = \frac{24}{\sqrt{3}}$$

Now that we have $y$, we can find $x$ by using tangent: $\tan(60°) = \dfrac{12/\sqrt{3}}{x}$, which gives

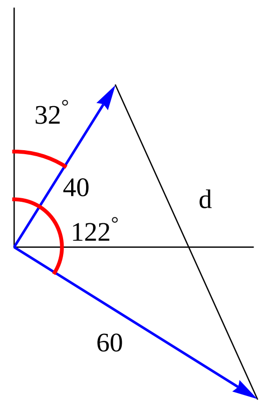
$$\sqrt{3} = \frac{12/\sqrt{3}}{x} \rightarrow x = 4$$

Lastly, all we need to determine is the length $h_2$. To do this, we will use $\cos(60°) = \dfrac{x}{h_2}$:

$$\frac{1}{2} = \frac{4}{h_2} \longrightarrow h_2 = 8$$

8. Two ships leave a port at the same time, with the first sailing on a bearing of 32° at 16 knots, and the second on a bearing of 122° at 24 knots. How far apart are they after 2.5 hours? (Knots are nautical miles/hour)

First, we draw a picture, including distances traveled after 2.5 hours.



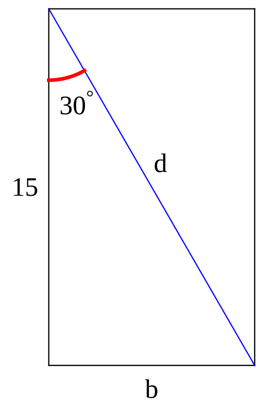

Next, we know the angle formed between the two boats (the angle with vertex at origin with sides that terminate at the boats), is 90°. We can now use the Pythagorean Theorem:

$$d^2 = 40^2 + 60^2$$

which gives $d = \sqrt{5200} = 20\sqrt{13}$ nautical miles.

9. One side of a rectangle measures 15cm. The angle between the diagonal and that side is 30°. Find the length of the unknown side and the diagonal.

A picture will help illustrate this problem:



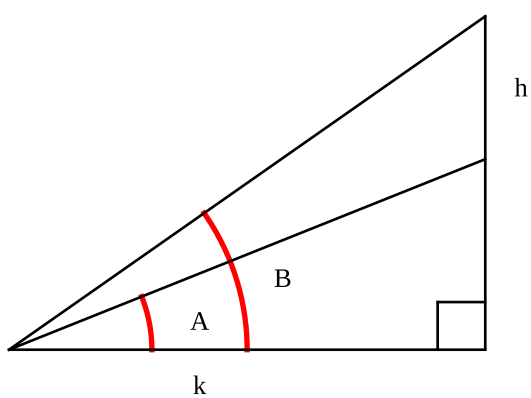

From this, we see that

$$\tan(30°) = \frac{b}{15} \longrightarrow x = 5\sqrt{3}$$

and

$$\cos(30°) = \frac{15}{d} \longrightarrow d = 10\sqrt{3}$$

10. Find a formula for $h$ in terms of $k$, $A$, and $B$. You may assume that $A < B$. (Here $h$ is the height of the upper triangle, not the entire triangle.)




If $d$ is the second distance on the right side, then note that

$$\tan(A) = \frac{d}{k} \text{ and } \tan(B) = \frac{h+d}{k}$$

From these two, we get the following:

$$d = k\tan(A) \text{ and } d = k\tan(B) - h$$

Setting the right hand sides equal to each other gives

$$k\tan(A) = k\tan(B) - h,$$

and solving for $h$ gives

$$h = k(\tan(B) - \tan(A))$$