Math 2215 - Calculus 1

Quiz #12 - 2011.04.18

Solutions

Find the finite area between the curves $f(x) = x^3 - 1$ and $g(x) = x - 1$.

First, we set $f(x) = g(x)$, which gives $x^3 - 1 = x - 1$, or $x^3 = x$. The solutions to this are $x = -1$, $x = 0$, and $x = 1$. Thus, the area can be computed by

$$\begin{aligned}
A &= \int_{-1}^{1} |f(x) - g(x)| \, dx \\
&= \int_{-1}^{0} f(x) - g(x) \, dx + \int_{0}^{1} g(x) - f(x) \, dx \\
&= \int_{-1}^{0} x^3 - x \, dx + \int_{0}^{1} x - x^3 \, dx \\
&= 2 \int_{-1}^{0} x^3 - x \, dx \\
&= 2 \left[\frac{1}{4}x^4 - \frac{1}{2}x^2 \right] \Big|_{-1}^{0} \\
&= 2 \left[0 - \left(\frac{1}{4} - \frac{1}{2} \right) \right] \\
&= 2 \cdot \frac{1}{4} \\
&= \frac{1}{2}
\end{aligned}$$