Math 2283 - Introduction to Logic

Quiz #4 - 2010.09.10

Solutions

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1. Construct a truth table for the following statement: $\sim (p \to q) \wedge \sim (q \to p)$

p	q	$p \to q$	$\sim (p \to q)$	$q \to p$	$\sim (q \to p)$	$\sim (p \to q) \wedge \sim (q \to p)$
T	T	T	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	F	T	F	F

2. State the three conjugate sentences (label each one appropriately) to the following conditional statement:

Conditional: If n is even, then n^2 is even.

Converse: If n^2 is even, then n is even.

Inverse: If n is not even, then n^2 is not even. (Or: If n is odd, then n^2 is odd.)

Contrapositive: If n^2 is not even, then n is not even. (Or: If n^2 is odd, then n is odd.)

3. Note that all four conditional statements from problem 2 are true. Explain why this fact, along with the truth table from problem 1 prove that the negation of a conditional statement cannot, itself, be a conditional statement of the form $a \to b$.

If the negation of a conditional statement was of the form $a \to b$, and we take an instance in which the three corresponding conjugate statements are also true, then the negations of all four of these statements must be false. However, from the truth table in problem 1, it is not true that a conditional and corresponding converse can both be false at the same time. Thus we must exclude the possibility that the negation of a conditional statement is can be expressed in the form $a \to b$.