Math 4133 - Linear Algebra

Homework #2

Assigned - 2011.01.26

Name: _____

Please answer, as fully as possible, all of the following questions. For those problems requiring *Maple*, please email me the *Maple* worksheet that goes along with the question. You do NOT have to print out the *Maple* worksheets.

1. Give both an algebraic and geometric explanation as to why underdetermined systems typically have an infinite number of solutions while overdetermined systems typically have no solution.

2. Using your answer to problem 1, construct (if possible) underdetermined and overdetermined systems that have a single solution, that is, a solution of dimension zero.

3. In Example 3.2.4, two elementary matrices were combined, e.g. E_7 and E_6 were replaced with one matrix, E_{76}. Explain how this process can be generalized to create modified elementary matrices which zero out all entries in a column except the diagonal entry. What *Maple* command does this process automatically?

4. Consider the following system of equations:

$$\begin{aligned} 2x_1 + 3x_2 - 4x_3 + x_4 + 4x_5 &= 5 \\ -6x_1 + 7x_2 + 8x_3 - 2x_4 + 3x_5 &= -2 \\ 9x_2 + 7x_3 + 8x_4 + 2x_5 &= 3 \\ -7x_1 + 3x_2 + 7x_3 + 2x_4 + 7x_5 &= 0 \end{aligned}$$

(a) Perform Gauss-Jordan elimination on the system of equations.

(b) Write out the solutions to this system, and give the dimension of the solution and the space $\mathbb{R}^n$ it lies in.

(c) Write out the solutions to this system in column matrix format using scalar multiplication by the independent variables.

5. Given the following three systems of equations

a) $\begin{aligned} 3x + 5y &= 7 \\ 4x + 6y &= 0 \end{aligned}$ b) $\begin{aligned} 3x + 5y &= 12 \\ 4x + 6y &= -10 \end{aligned}$ c) $\begin{aligned} 3x + 5y &= -2 \\ 4x + 6y &= 2 \end{aligned}$

(a) Construct a matrix, which row reduced, will solve all three systems of equations simultaneously.

(b) Solve the all three systems of equations by row reducing the matrix created in part (a).