Math 4133 - Linear Algebra
Quiz #6 - 2011.02.21
Solutions

Solve the following system of equations by back substitution:

$$\begin{bmatrix} 2 & -1 & 4 & 2 \\ 0 & 3 & -1 & 2 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \\ 3 \end{bmatrix}$$

We use the last equation $-x_4 = 3$ to get $x_4 = -3$.

The second to last equation is $2x_3 - x_4 = 1$. However, since $x_4 = -3$, we get $2x_3 + 3 = 1$, or $x_3 = -1$. The second equation is $3x_2 - x_3 + 2x_4 = -2$. Into this, we substitute $x_4 = -3$ and $x_3 = -1$, which gives $3x_2 + 1 - 6 = -2$ and solving for x_2 gives $x_2 = 1$. We now only have to solve for x_1, which is the first equation: $2x_1 - x_2 + 4x_3 + 2x_4 = 1$. Since we know x_2, x_3 and x_4 all we have left in this equation is x_1: $2x_1 - 1 - 4 - 6 = 1$ which yields $x_1 = 6$.

Our solution is given by $\{x_1 = 6, x_2 = 1, x_3 = -1, x_4 = -3\}$.