Math 1613 - Trigonometry

Quiz #10 - 2009.10.20

Solutions

Using just the following two identities:

$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$
$$\cos(x) = \sin\left(\frac{\pi}{2} - x\right),$$

prove the following identity.

1. $\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$

Setting $a = \frac{\pi}{2}$ and $b = x$ in the first equation yields:

$$\cos(\frac{\pi}{2} - x) = \cos(\frac{\pi}{2})\cos(x) + \sin(\frac{\pi}{2})\sin(x),$$

where $\cos(\frac{\pi}{2}) = 0$ and $\sin(\frac{\pi}{2}) = 1$, therefore

$$\begin{aligned}\cos(\frac{\pi}{2} - x) &= \cos(\frac{\pi}{2})\cos(x) + \sin(\frac{\pi}{2})\sin(x) \\ &= 0 \cdot \cos(x) + 1 \cdot \sin(x) \\ &= \sin(x)\end{aligned}$$

2. Using the first two identities and the identity from problem 1, show:

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

We start with $\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$, and let $x = a + b$:

$$\begin{aligned}\sin(a+b) &= \cos\left(\frac{\pi}{2} - (a+b)\right) \\ &= \cos\left(\left(\frac{\pi}{2} - a\right) - b)\right).\end{aligned}$$

Using the first given equation with $a = \frac{\pi}{2} - a$ and $b = b$ gives

$$\begin{aligned}\cos\left(\left(\frac{\pi}{2} - a\right) - b)\right) &= \cos\left(\frac{\pi}{2} - a\right)\cos(b) + \sin\left(\frac{\pi}{2} - a\right)\sin(b) \\ &= \sin(a)\cos(b) + \cos(a)\sin(b)\end{aligned}$$