Math 3113 - Multivariable Calculus

Homework #4 - 2008.02.06

Due Date - 2008.02.13

Solutions

Find an equation of the form $r = f(\theta, z)$ in cylindrical coordinates for the following surfaces.

1. $x^2 + y^2 + z^2 = 4$

$$\begin{aligned} x^2 + y^2 + z^2 &= 4 \\ r^2 + z^2 &= 4 \end{aligned}$$

Solving for r gives

$$r = \sqrt{4 - z^2}.$$

2. $x^2 - y^2 = 4$

$$\begin{aligned} x^2 - y^2 &= 4 \\ r^2\cos^2(\theta) - r^2\sin^2(\theta) &= 4 \\ r^2\left(\cos^2(\theta) - \sin^2(\theta)\right) &= 4 \end{aligned}$$

Solving for r gives

$$r = \frac{2}{\sqrt{\cos^2(\theta) - \sin^2(\theta)}} = \frac{2}{\sqrt{\cos(2\theta)}}.$$

3. $z = 3xy$

$$\begin{aligned} z &= 3xy \\ z &= 3r^2\cos(\theta)\sin(\theta) \end{aligned}$$

Solving for r gives

$$r = \sqrt{\frac{z}{3\cos(\theta)\sin(\theta)}} = \sqrt{\frac{2z}{3\sin(2\theta)}}.$$

Find an equation of the form $\rho = f(\theta, \phi)$ in spherical coordinates for the following surfaces.

4. $z = 2$

$$\begin{aligned} z &= 2 \\ \rho\cos(\phi) &= 2 \end{aligned}$$

Solving for ρ gives

$$\rho = \frac{2}{\cos(\phi)}.$$

5. $x = z^2$

$$\begin{aligned} x &= z^2 \\ \rho\cos(\theta)\sin(\phi) &= \rho^2\cos^2(\phi) \\ \cos(\theta)\sin(\phi) &= \rho\cos^2(\phi) \end{aligned}$$

Solving for ρ gives

$$\rho = \frac{\cos(\theta)\tan(\phi)}{\cos(\phi)}.$$

6. $x^2 - y^2 = 4$

$$\begin{aligned} x^2 - y^2 &= 4 \\ \rho^2\sin^2(\phi)\cos^2(\theta) - \rho^2\sin^2(\phi)\sin^2(\theta) &= 4 \\ \rho^2\sin^2(\phi)\left(\cos^2(\theta) - \sin^2(\theta)\right) &= 4 \\ \rho^2\sin^2(\phi)\cos(2\theta) &= 4 \end{aligned}$$

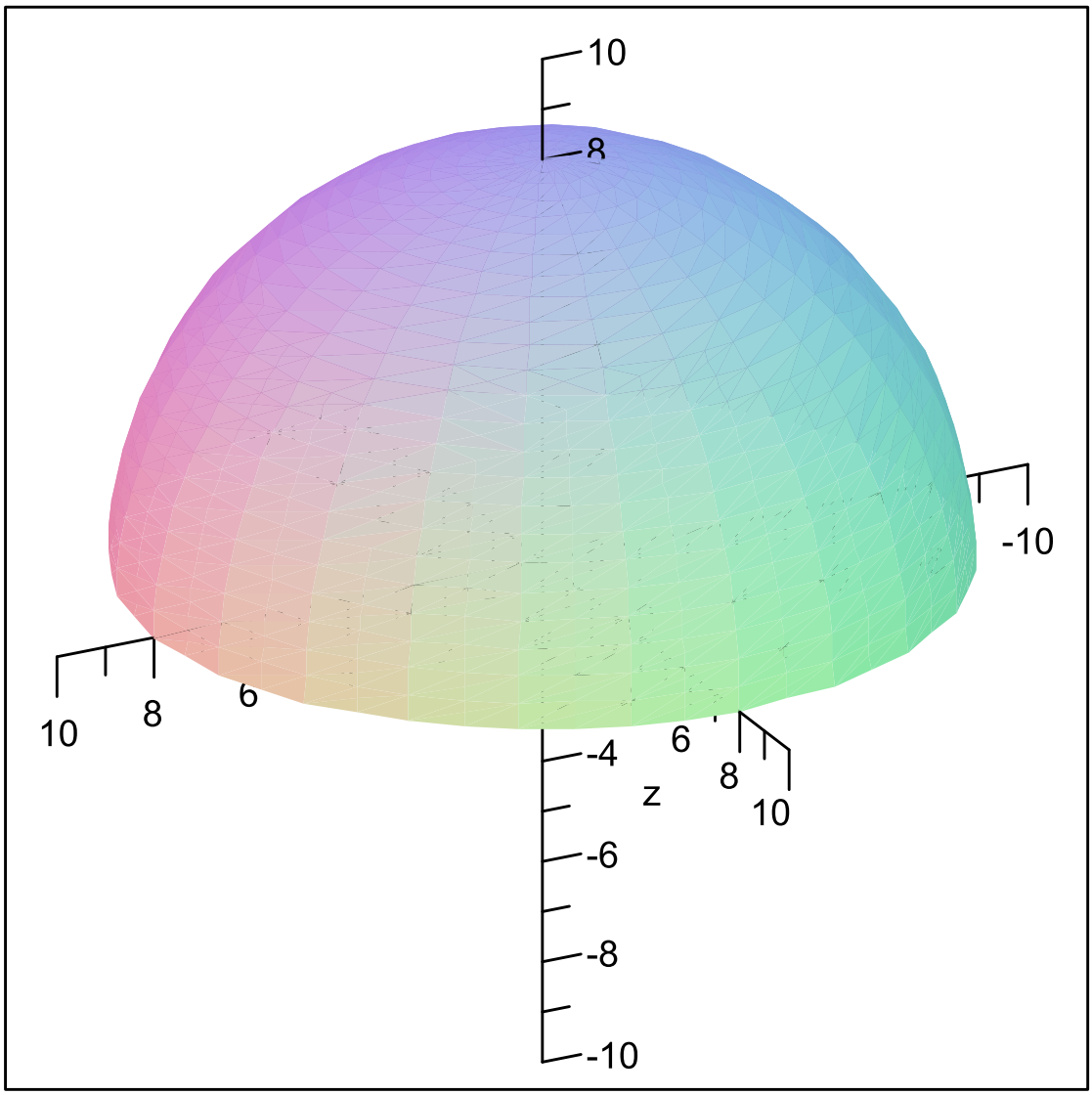
Solving for ρ gives

$$\rho = \frac{2}{\sin(\phi)}\frac{1}{\sqrt{\cos(2\theta)}}.$$

Sketch the graphs of the following sets of points in spherical coordinates.

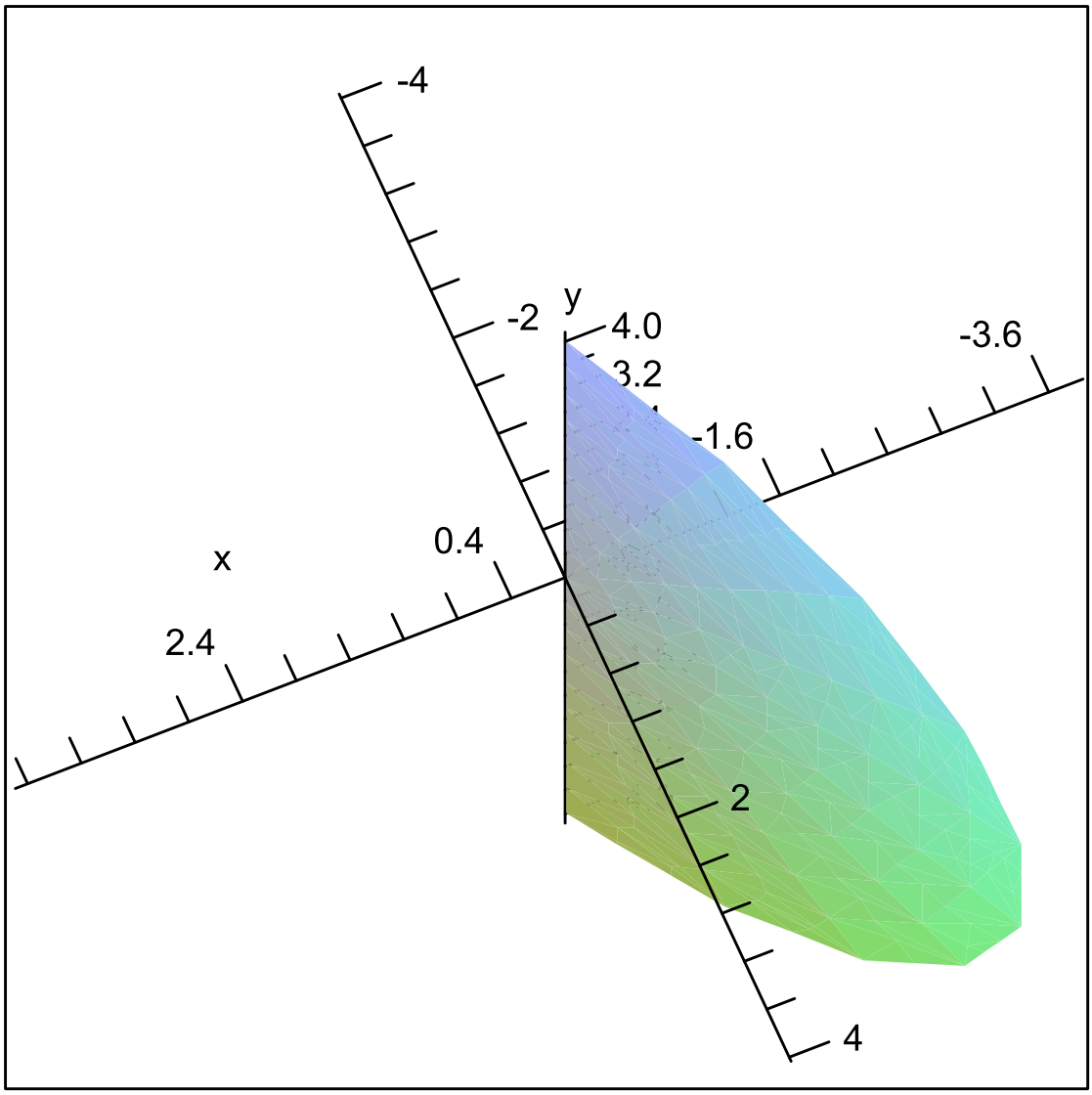
7. $\rho = 8, \quad 0 \leq \theta \leq 2\pi, \quad 0 \leq \phi \leq \frac{\pi}{2}$

This is the upper half sphere of radius 8.



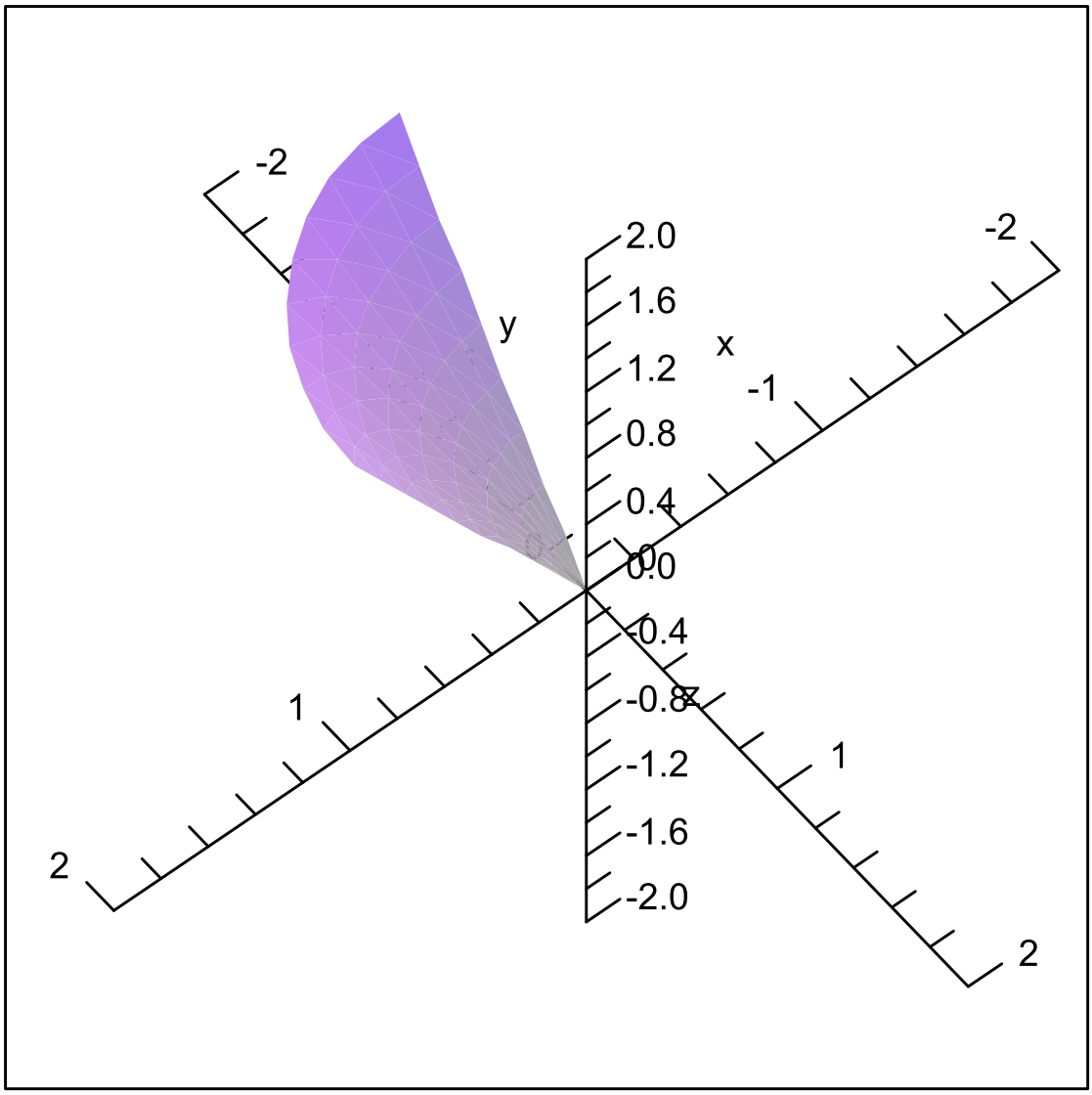
8. $\theta = \frac{2\pi}{3}, \quad 0 \leq \rho \leq 4, \quad 0 \leq \phi \leq \pi$

This is the disc of radius 4 centered on the z-axis which forms an angle of $\theta = \frac{2\pi}{3}$ in the xy-plane.



9. $\phi = \frac{\pi}{6}, \quad 0 \leq \rho \leq 2, \quad \frac{3\pi}{2} \leq \theta \leq 2\pi$

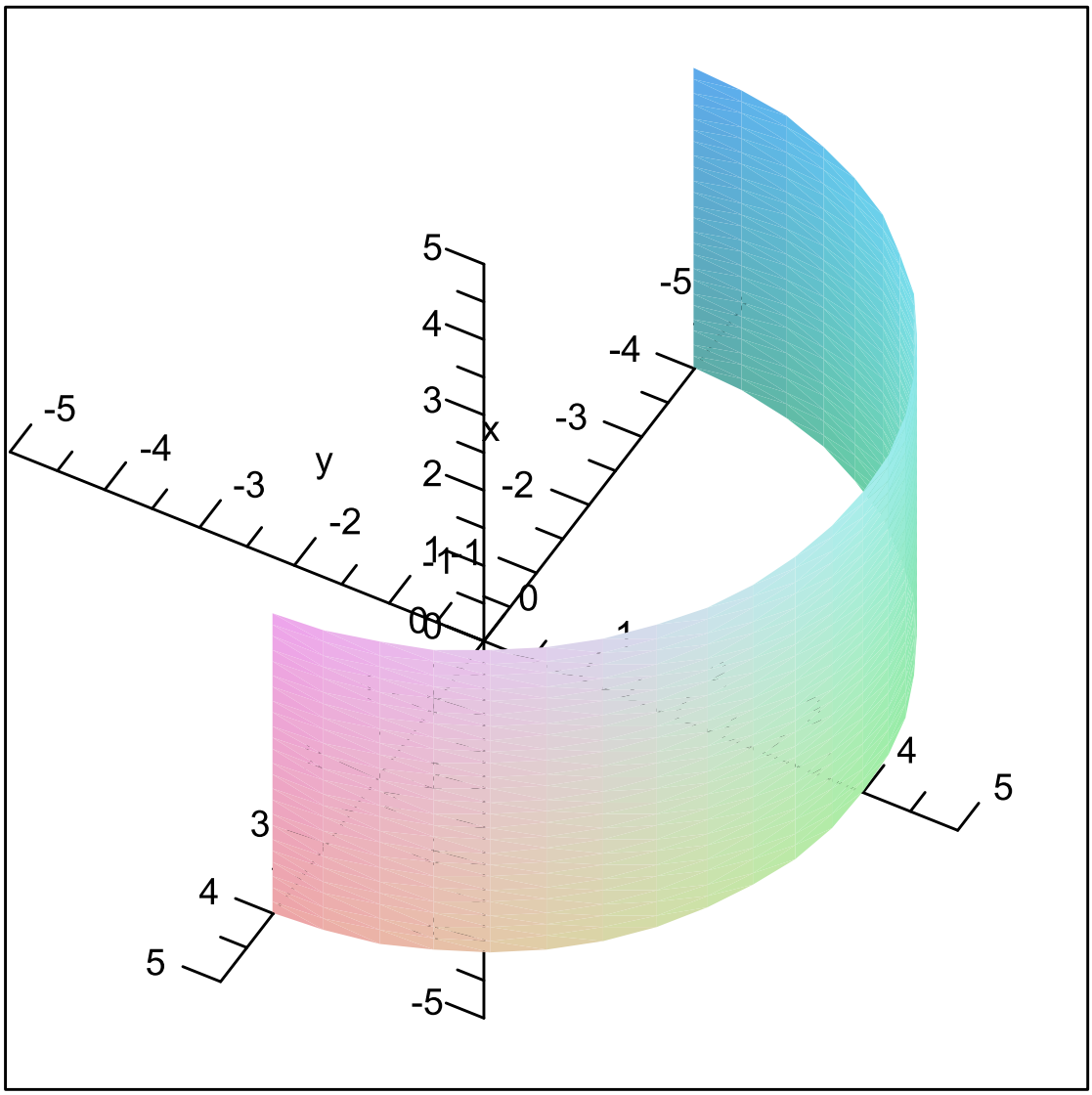
This is $\frac{1}{4}$ portion of the upper cone located in the octant with negative y and positive x coordinates.



Sketch the graphs of the following sets of points in cylindrical coordinates.

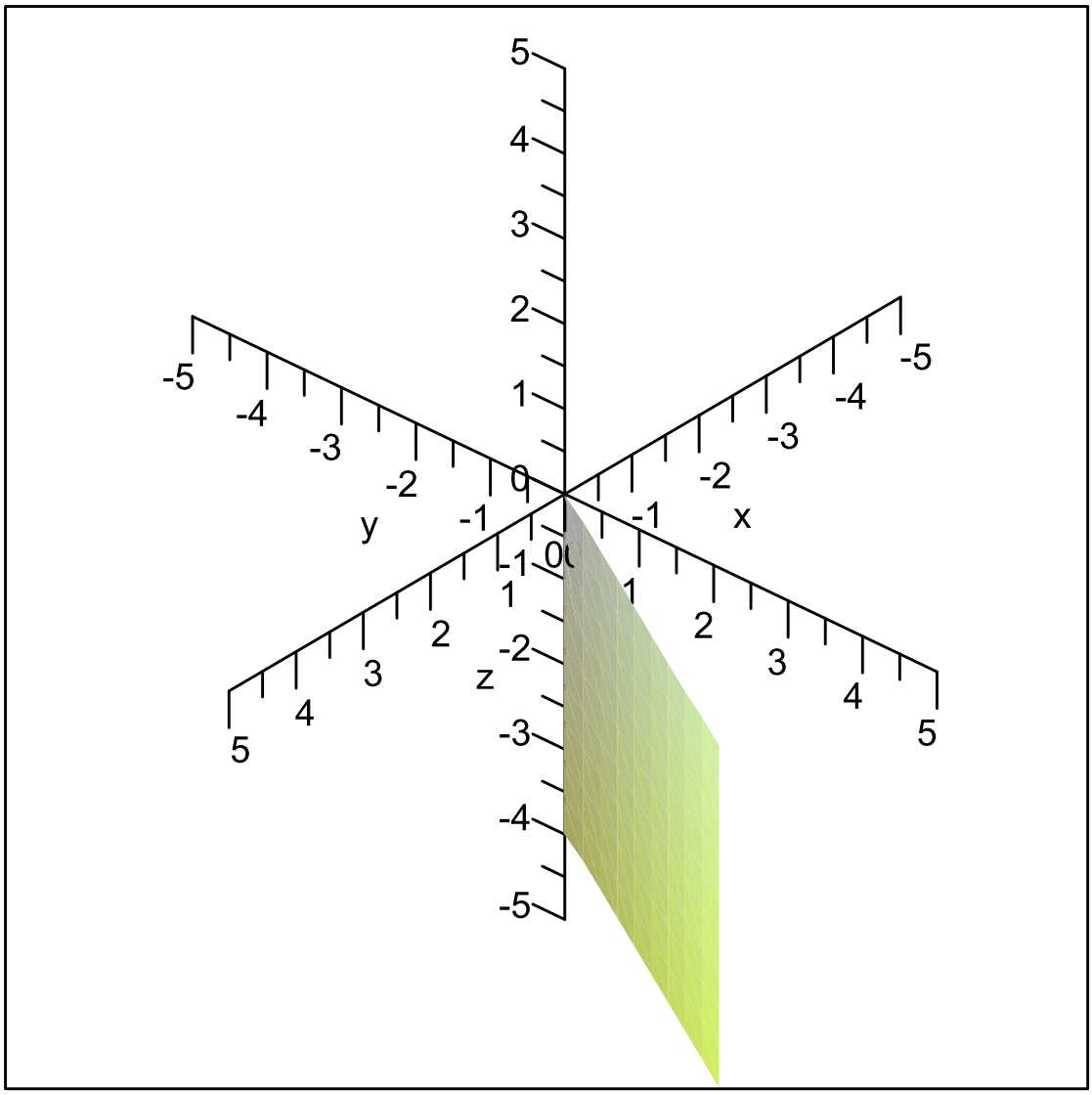
10. $r = 4, \quad 0 \leq \theta \leq \pi, \quad 0 \leq z \leq 4$

This is half a cylinder of radius 4 with positive y-coordinates and height 4 starting in the xy-plane.



11. $\theta = \frac{\pi}{3}, \quad 0 \leq r \leq 5, \quad -4 \leq z \leq 0$

This is a plane of height 4 starting at $z = -4$ which is 5 units long and forms an angle of $\theta = \frac{\pi}{3}$ with the xy-plane.



12. $z = -2, \quad 0 \le r \le 5, \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$

This is a half disc of radius 5 located 5 units below the xy-plane with negative x-coordinates.