

Math 2143 - Brief Calculus with Applications

Quiz #8 - 2006.03.08

Solutions

Consider the function $f(x) = x^3 - x^4$. Answer the following questions.

1. Find all the roots of $f(x)$.

$f(x)$ is zero when $x = 0$ and $x = 1$.

2. Find $f'(x)$.

$$f'(x) = 3x^2 - 4x^3 = x^2(3 - 4x)$$

3. Find all the critical points of $f(x)$.

Critical points are at $x = 0$ and $x = \frac{3}{4}$.

4. State the intervals of increase and decrease of $f(x)$.

Notice that since x^2 is always positive, the sign of $f'(x) = 3x^2 - 4x^3 = x^2(3 - 4x)$ is determined only by the term $(3 - 4x)$, which is negative for $x > \frac{3}{4}$ and positive for $x < \frac{3}{4}$. Therefore, the intervals of increase are $(-\infty, 0)$ and $(0, \frac{3}{4})$ while the interval of decrease is $(\frac{3}{4}, \infty)$.

5. Find all the relative extrema of $f(x)$.

From the previous problem, we see that the point $x = 0$ is not a relative extrema, but $x = \frac{3}{4}$ is a relative maximum.

6. Sketch a graph of $f(x)$.

