Math 2215 - Calculus 1

Homework #3 - 2005.09.13

Due Date - 2005.09.19

Solutions

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1. Prove that if $f(x)$ is even, then $f'(x)$ is odd.

Remember that $f'(x) = \lim_{w\to x} \frac{f(w)-f(x)}{w-x}$. Now, replacing x with $-x$ gives

$$f'(-x) = \lim_{-w\to -x} \frac{f(-w)-f(-x)}{-w+x}.$$

Using the fact that $f(-x) = f(x)$, one now has

$$= \lim_{-w\to -x} \frac{f(w)-f(x)}{-w+x}$$

$$= \lim_{-w\to -x} -\frac{f(w)-f(x)}{w-x} = -f'(x).$$

2. Find the points on the curve $y = ax^3 + bx^2 + cx + d$ where the tangent is horizontal. Here, assume that $a \neq 0$.

$$y' = 3ax^2 + 2bx + c$$

Setting $y' = 0$ and solving for x will give all the x values where a horizontal tangent line exists. The quadratic formula can be used, which gives

$$x = \frac{-2b \pm \sqrt{4b^2 - 12ac}}{6a}$$

3. From your answer in problem 2, what guarantees that the function y has at least one point at which the tangent line is horizontal.

For the values of x found above to be valid, one must enforce

$$4b^2 - 12ac \geq 0$$

or

$$b^2 \geq 3ac.$$

4. What would it mean about the shape of the graph of y if there was only ONE point at which there was a horizontal tangent line.

If there was only one horizontal tangent, the the graph of the cubic must be always increasing or decreasing.

5. Use the product rule to find $\frac{d}{dx}\left[f(x)\cdot g(x)\cdot h(x)\cdot i(x)\right]$.

$$= f'(x)\cdot g(x)\cdot h(x)\cdot i(x) + f(x)\cdot g'(x)\cdot h(x)\cdot i(x) + f(x)\cdot g(x)\cdot h'(x)\cdot i(x) + f(x)\cdot g(x)\cdot h(x)\cdot i'(x)$$

6. Use the product rule to show that $\frac{d}{dx}\left[g(x)^n\right] = n\left(g(x)^{n-1}\right)g'(x)$.

Here, we write

$$\frac{d}{dx}\left[g(x)^n\right] = \frac{d}{dx}\left(g(x)\cdot g(x)\cdots g(x)\right)$$

$$= g'(x)\cdot\left(g(x)^{n-1}\right) + g(x)\cdot g'(x)\cdot\left(g(x)^{n-2}\right) + \ldots + \left(g(x)\right)^{n-1} g'(x)$$

notice that after rearranging the products in each term, they are all of the form

$$g'(x)\cdot\left(g(x)^{n-1}\right),$$

furthermore, there are exactly n terms, which gives

$$\frac{d}{dx}\left[g(x)^n\right] = n\left(g(x)^{n-1}\right)g'(x).$$

7. If $f(2) = 3$, $g(2) = 1$, $f'(2) = 2$ and $g'(2) = 4$, find the following values.

a) $\left(\frac{f}{g}\right)'(2)$

$$\left(\frac{f}{g}\right)'(2) = \frac{f'(2)g(2) - f(2)g'(2)}{g(2)^2} = -10$$

b) $\left(\frac{f+g}{f-g}\right)'(2)$

$$\left(\frac{f+g}{f-g}\right)'(2) = \frac{\left(f'(2)+g'(2)\right)\left(f(2)-g(2)\right) - \left(f(2)+g(2)\right)\left(f'(2)-g'(2)\right)}{\left(f(2)-g(2)\right)^2} = 5$$

c) $\left[(f+2g)(f-3g)\right]'(2)$

$$\left[(f+2g)(f-3g)\right]'(2) = \left(f'(2)+2g'(2)\right)\left(f(2)-3g(2)\right) + \left(f(2)+2g(2)\right)\left(f'(2)-3g'(2)\right) = -50$$

8. Compute the derivatives of the following functions.

a) $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots + a_nx^n$

$$p'(x) = a_1 + 2a_2x + 3a_3x^2 + \ldots + na_nx^{n-1}$$

b) $t(u) = \frac{\sin(u)}{u^2+\cos(u)}$

$$t'(u) = \frac{\cos(u)\left(u^2+\cos(u)\right) - \sin(u)\left(2u - \sin(u)\right)}{\left(u^2+\cos(u)\right)^2}$$

$$= \frac{1 + u^2\cos(u) - 2u\sin(u)}{\left(u^2+\cos(u)\right)^2}$$

c) $g(r) = r^2e^r\left(\cos(r)+\sin(r)\right)$

$$g'(r) = 2re^r\left(\cos(r)+\sin(r)\right) + r^2e^r\left(\cos(r)+\sin(r)\right) + r^2e^r\left(\cos(r)-\sin(r)\right)$$

9. Evaluate the following limit: $\lim_{x\to1}\frac{x^{1252352}-1}{x-1}$

Notice by the form of the limit that we have

$$\left[\frac{d}{dx}x^{1252352}\right]\Bigg|_{x=1} = \lim_{x\to1}\frac{x^{1252352}-1}{x-1} = 1252352$$