

Math 2283 - Introduction to Logic

Final Exam - 2006.12.03

Due Date - 2006.12.13 by 10:00 AM

Name: _____

Please answer each question as fully as possible. Partial credit will only be given if you show your work. Also, make sure your work is neat and orderly. Finally, DO NOT work together, DO NOT get assistance from anyone but me!

Disclaimer: *All characters appearing in this work are fictitious. Any resemblance to real persons, living or dead, is purely coincidental.*

1. Consider the function $f(x, y) = 4x^2 - 4xy + y^2$, where x and y are any real numbers. Determine if the following statements are true or false.

a) $\exists x \exists y f(x, y) = 0$

b) $\forall x \exists y f(x, y) = 0$

c) $\forall y \exists x f(x, y) = 0$

d) $\exists x \exists y f(x, y) > 0$

e) $\exists x \exists y f(x, y) < 0$

2. Convert the following sentential wffs to polish sentential.

a) $((\neg R \Leftrightarrow (P \wedge Q)) \Rightarrow ((\neg R \vee \neg P) \wedge (R \vee \neg Q)))$

b) $((\neg((A \Rightarrow B) \vee (B \Rightarrow \neg A)) \vee B) \Rightarrow (C \wedge \neg D))$

3. Construct a sentential wff which, when converted to polish sentential, has at least 5 of the same connectives in a row at the beginning of the wff.

4. Construct a fifth premise, with at least one connective, which will make the following argument valid.

$$(\neg R \Rightarrow P)$$

$$(Q \Rightarrow (R \vee P))$$

$$((R \wedge Q) \Leftrightarrow \neg P)$$

$$(P \vee (R \wedge Q))$$

Premise 5 goes here

$$\therefore ((P \wedge \neg Q) \Rightarrow R)$$

5. Consider a language with predicates Lxy , Gxy and Hx and a name c . Define the following structures:

Structure S1:

Domain = $\{0, 1, 2, \dots, 20\}$

Lxy is $x < y$

Gxy is $x + y = 12$

Hx is $x < 12$

c is 10

Structure S2:

Domain = $\{-20, -19, \dots, 19, 20\}$

Lxy is $x \leq y$

Gxy is $x + y = 12$

Hx is $x < 12$

c is 20

Structure S3:

Domain = $\{-30, -29, \dots, 29, 30\}$

Lxy is $x < y$

Gxy is $x + y = 12$

Hx is $|x| < 12$

c is 25

Evaluate each of the following wffs for each structure.

a) $\forall x \forall y ((Lxy \wedge Gxy) \Rightarrow Hx)$

b) $\forall x \exists y ((Lxy \wedge Gxy) \Rightarrow Hy)$

c) $\forall y ((Lyc \wedge Gcy) \Rightarrow Hy)$

e) $\exists y ((Lyc \wedge Gcy) \Rightarrow Hy)$

6. Construct truth tables for the following sentential wffs.

a) $((P \wedge \neg Q) \Rightarrow (R \vee (R \Leftrightarrow P)))$

b) $((\neg P \Rightarrow Q) \Leftrightarrow (R \vee (P \wedge \neg Q)))$

c) $\neg((S \wedge (P \vee Q)) \vee (P \Rightarrow Q))$

7. Consider a formal language over the domain of students in both Honors Logic courses where:

Hxy is x is smarter than y

Txy is x talks more than y

Kx is x is a knight

Sy is y is silly

Tx is x is a Texan

k denotes Keith W.

j denotes John S.

Formalize the following sentences.

a) All Texans are silly.

b) Every knight is silly.

c) Some silly knights are smarter than all not silly Texans.

d) John S. talks more than Keith W.

e) Not every Texan is a knight.

f) Not anyone who is a silly Texan is smarter than John S.

g) If someone talks more than Keith W., then anyone is smarter than all knights.

8. Determine if each of the following arguments are valid or invalid. If invalid, construct a universe to demonstrate the fact. If valid, do not prove by truth tables, prove by the method of natural deduction. In your proofs, be sure to include the validation of each step.

a)

$$\neg P$$

$$\therefore P \Rightarrow \neg P$$

b)

$$P \vee Q$$

$$\neg P$$

$$\therefore Q$$

c)

$$\neg \forall x Fx$$

$$\therefore \forall x \neg Fx$$

d)

$$\forall x (Px \Rightarrow Fx)$$

$$\exists x Px$$

$$\therefore \exists x Fx$$

e)

$$\forall x (Px \Leftrightarrow Fx)$$

$$\exists x Px$$

$$\therefore \exists x Fx$$

9. Negate the following statements. Do not simply place “It is not true that” in front of the statement.

a) Everyone likes kittens and Sunday dinner.

b) Not anyone can look at the sun.

c) If someone attempts to answer all the problems on this exam, then every kitten’s life will be saved.

d) Some people can read.

e) All unicorns can read.

10. Who had nearly fought the Dragon of Angnor, had nearly stood up to the vicious Chicken of Bristol and had personally wet himself at the Battle of Badon Hill?