Math 2283 - Introduction to Logic

Homework #5 - 2006.10.12

Due Date - 2006.10.18

Solutions

Determine if the following arguments are valid or invalid. Do not use a truth table to determine your answer, instead use the shortcut we have recently discussed. If the argument is valid, state why (be sure to cover all possibilities). If the argument is invalid, state why and also state the conditions on the atomic wff's which make it invalid.

1.
$((P \wedge R) \Rightarrow Q)$
$(Q \Rightarrow R)$
$(Q \vee R)$
$\therefore (P \Leftrightarrow Q)$

The argument is invalid since the combination $P = F$, $Q = T$ and $R = T$ gives a set of true premises and a false conclusion.

2.
$((P \wedge R) \Rightarrow Q)$
$(Q \Rightarrow R)$
$(Q \vee R)$
$\therefore (P \Rightarrow Q)$

The argument is valid. The only time the conclusion is false is when $P = T$ and $Q = F$. Using the third premise, we see that $R = T$, but with this combination, the first premise is false.

3.
$((P \vee R) \Rightarrow S)$
$((S \wedge R) \Leftrightarrow P)$
$(\neg P \Rightarrow (\neg S \vee R))$
$\therefore (R \vee S)$

The argument is invalid. The combination $R = F$, $S = F$ and $P = F$ yields true premises and a false conclusion.

4.
$((P \vee R) \Rightarrow S)$
$((S \wedge R) \Leftrightarrow P)$
$(\neg P \Rightarrow (S \vee R))$
$\therefore (R \vee S)$

This argument is valid. To make the conclusion false, $R = F$ and $S = F$. This forces $P = F$ via premise 1. However this makes premise 3 false.

5.
$((R \vee S) \Rightarrow (P \wedge Q))$
$((S \wedge R) \Leftrightarrow \neg Q)$
$(\neg S \vee (Q \Rightarrow R))$
$\therefore ((P \wedge Q) \Rightarrow R)$

The argument is invalid since the combination $P = T$, $Q = T$, $R = F$ and $S = F$ yields true premises and a false conclusion.

6.
$((R \vee S) \Rightarrow (P \wedge Q))$
$((S \wedge R) \Leftrightarrow \neg Q)$
$((P \wedge S) \Leftrightarrow (R \vee Q))$
$(\neg S \vee (Q \Rightarrow R))$
$\therefore ((P \wedge Q) \Rightarrow R)$

This argument is valid. To make the conclusion false, $R = F$, $P = T$ and $Q = T$. This forces $S = T$ via premise 4. However this makes premise 3 false.