

Math 2283 - Introduction to Logic

Exam #1 - 2017.02.01 Solutions

1. Determine whether each of the following are constants, sentential functions, designatory functions, or sentences.

(a) x is the largest city in Oklahoma.

sentential function

(b) The largest city in Oklahoma.

constant

(c) City x has a larger population than city y .

sentential function

(d) The state capitol of x .

designatory function

(e) $\forall x$, if x is a state, then x has a capitol.

sentence

2. Determine which of the following quantified sentences are true, and which are false. Here, you may assume that x and y belong to the set of people in this classroom.

(a) $\exists x, y$ x is at least as old as y .

This is a true sentence, we can find two people who are of a different age, this one will be at least as old as the other.

(b) $\exists x \forall y$ x is at least as old as y .

This is true, there is an oldest person (who is at least as old as themselves).

(c) $\forall y \exists x$ x is at least as old as y .

This is true since (b) was true.

(d) $\forall x, y$ x is at least as old as y .

This is false, just pick x which is older than y , then swap them, it must still hold, which clearly it does not.

3. For what values is the following sentential function false?

$\exists y$ y is a business day after day x of a standard week.

This does not hold for x being Friday and Saturday.

4. For the following conditional sentences, state its (a) inverse, (b) converse, and (c) contrapositive.

If there is a state revenue failure, then funding for state agencies will be reduced.

- (a) If there is no state revenue failure, then funding for state agencies will not be reduced.
 (b) If funding for state agencies is reduced, then there is a state revenue failure.
 (c) If the funding for state agencies is not reduced, then there will be no state revenue failure.

5. State the Rule of Detachment.

If two sentences are accepted as true, of which one has the form of an implication while the other is the antecedent of this implication, then that sentence may also be recognized as true, which forms the consequent of the implication. (We *detach* thus, so to speak, the antecedent from the whole implication.)

6. State the Rule of Substitution.

If a sentence of a universal character, that has already been accepted as true, contains sentential variables, and if these variables are replaced by other sentential variables or by sentential functions or sentences – always substituting equal expressions for equal variables throughout –, then the sentence obtained in this way may be recognized as true.

7. State the Law of Contraposition.

Any conditional sentence implies its corresponding contrapositive sentence, and may be formulated symbolically as:

$$(p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$$

8. Construct a truth table for the sentence $(p \vee q \vee r) \leftrightarrow [\sim p \rightarrow (q \vee r)]$.

p	q	r	$p \vee q$	$p \vee q \vee r$	$\sim p$	$q \vee r$	$\sim p \rightarrow (q \vee r)$	$(p \vee q \vee r) \leftrightarrow [\sim p \rightarrow (q \vee r)]$
T	T	T	T	T	F	T	T	T
T	T	F	T	T	F	T	T	T
T	F	T	T	T	F	T	T	T
T	F	F	T	T	F	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	F	T	T	T	T	T
F	F	F	F	F	T	F	F	T

9. Convert the following to logical symbolism:

If it is not that p or it is not the case that q , then it is not the case that: p and q .

This should read as $(\sim p \vee \sim q) \rightarrow \sim (p \wedge q)$.

10. Justify each of steps (4) – (7) in the following proof.

(1)	$\sim q$	Assumption
(2)	$p \vee q$	Assumption
(3)	$(p \vee q) \rightarrow (\sim p \rightarrow q)$	Logical Law
(4)	$\sim p \rightarrow q$	Rule of Detachment on (2) and (3)
(5)	$(\sim p \rightarrow q) \rightarrow (\sim q \rightarrow p)$	Law of Contraposition, $p : \sim p$
(6)	$\sim q \rightarrow p$	Rule of Detachment on (4) and (5)
(7)	p	Rule of Detachment on (1) and (6)