Math 2315 - Calculus 2

Quiz #4 - 2017.01.17

Solutions

Compute the following integral: $\int x\, e^x \cos(x)\, \mathrm{d}x$

We do integration by parts with $f = x$ and $g' = e^x \cos(x)$. However, in order to do this, we would need to know what g is, but we have computed the antiderivative of $e^x \cos(x)$ before, it is $g = \frac{1}{2}e^x(\cos(x) + \sin(x))$.

$$\begin{aligned}
\int x\, e^x \cos(x)\, \mathrm{d}x &= \frac{1}{2}x\, e^x(\cos(x) + \sin(x)) - \frac{1}{2}\int e^x(\cos(x) + \sin(x))\, \mathrm{d}x \\
&= \frac{1}{2}x\, e^x(\cos(x) + \sin(x)) - \frac{1}{2}\int e^x \cos(x)\, \mathrm{d}x - \frac{1}{2}\int e^x \sin(x))\, \mathrm{d}x \\
&= \frac{1}{2}x\, e^x(\cos(x) + \sin(x)) - \frac{1}{4}e^x(\cos(x) + \sin(x)) - \frac{1}{2}\int e^x \sin(x))\, \mathrm{d}x
\end{aligned}$$

Similarly, by integration by parts, we have computed the antiderivative of $g' = e^x \sin(x)$ before, it is $g = \frac{1}{2}e^x(-\cos(x) + \sin(x))$. Thus

$$\begin{aligned}
\int x\, e^x \cos(x)\, \mathrm{d}x &= \frac{1}{2}x\, e^x(\cos(x) + \sin(x)) - \frac{1}{4}e^x(\cos(x) + \sin(x)) - \frac{1}{2}\int e^x \sin(x))\, \mathrm{d}x \\
&= \frac{1}{2}x\, e^x(\cos(x) + \sin(x)) - \frac{1}{4}e^x(\cos(x) + \sin(x)) - \frac{1}{4}e^x(-\cos(x) + \sin(x)) + \mathcal{C} \\
&= \frac{1}{2}x\, e^x(\cos(x) + \sin(x)) - \frac{1}{2}e^x \sin(x) + \mathcal{C} \\
&= \frac{1}{2}e^x\left[x\cos(x) + x\sin(x) - \sin(x)\right] + \mathcal{C}
\end{aligned}$$