Math 2315 - Calculus 2

Final Exam - 2021.05.03

Name: ______________________________

$$\sin^2(t)+\cos^2(t)=1,\quad 1+\cot^2(t)=\csc^2(t),\quad \tan^2(t)+1=\sec^2(t)$$

$$e^x=\sum_{k=0}^{\infty}\frac{1}{k!}x^k,\quad \sin(x)=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}x^{2k+1}$$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin(\theta)$	$\sqrt{\frac{0}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{4}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{0}{4}}$
$\cos(\theta)$	$\sqrt{\frac{4}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{0}{4}}$	$-\sqrt{\frac{1}{4}}$	$-\sqrt{\frac{2}{4}}$	$-\sqrt{\frac{3}{4}}$	$-\sqrt{\frac{4}{4}}$

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1. Compute the following integrals.

(a) $\displaystyle\int \frac{3x+2}{\sqrt{1-x^2}}\,\mathrm{d}x$

(b) $\displaystyle\int \cot^3(t)\csc^2(t)\,\mathrm{d}t$

(c) $\displaystyle\int \frac{-2w+4}{(w^2+1)(w-1)^2}\,\mathrm{d}w$

(d) $\displaystyle\int \frac{1}{(4-z^2)^{3/2}}\,\mathrm{d}z$

(d) $\displaystyle\int_1^{\infty} \frac{1}{x\sqrt{x^2-1}}\,\mathrm{d}x$

2. Determine whether the following series converge or diverge:

(a) $\displaystyle\sum_{k=1}^{\infty}\frac{e^k}{1+e^{2k}}$

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{(2n)!}{(n!)^2}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

(d) $\displaystyle\sum_{k=1}^{\infty} \frac{\ln(n)}{n - \ln(n)}$

3. Find all values of x for which the following series converge:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{3^n}(x+2)^n$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k\,2^k}(x+2)^k$

4. Find the Maclaurin Series for $F(x) = x^2 \sin(x)$ by starting with the Maclaurin series for $\sin(x)$.

5. Use the Maclaurin Series for e^x to compute the following limit: $\displaystyle\lim_{x\to\infty} x^2\left(e^{-1/x^2} - 1\right)$.

6. Consider the parametric equation $(x(t), y(t)) = \left(\cos^3(t), \sin^3(t)\right)$ for $t \in [0, 2\pi]$.

   (a) Sketch a graph of the curve for $t \in [0, 2\pi]$. Note: This is NOT a circle or ellipse.

   (b) Compute the slope of the tangent line to the curve as $t \to \dfrac{\pi}{2}^-$.

   (c) Compute the slope of the tangent line to the curve as $t \to \dfrac{\pi}{2}^+$.

   (d) Be sure that your results from parts (b) and (c) agree with your sketch from part (a).

7. Verify that the polar function $r = f(\theta) = -\dfrac{4}{\cos(\theta)}$ is the vertical line $x = -4$.

8. Consider the polar function $r = f(\theta) = \dfrac{1}{2} + \sin(\theta)$.

   (a) Sketch $f(\theta)$ in the xy-plane.

   (b) Compute $\frac{dy}{dx}$ when $\theta = \dfrac{\pi}{6}$.